

HARJUTUSÜLESANDED lk.57

Ülesanne 1

Arvuta raadiuse ja kesknurga järgi kaare pikkus ja sektori pindala, kui $\alpha = \frac{3\pi}{4}$ ja $r = 1$ m.

Kasutame kaare pikkuse leidmiseks valemit $l = r \cdot \alpha = \frac{3\pi}{4} \cdot 1 = \frac{3\pi}{4}$ (m) ja sektori pindala $S = \frac{r^2 \cdot \alpha}{2} = \frac{1 \cdot 3\pi}{2 \cdot 4} = \frac{3\pi}{8}$ (m^2).

Ülesanne 2

Kella minutiosuti on 8 cm pikk. Kui pika tee läbib minutiosuti 10 minuti jooksul, kui pika tee 25 minuti jooksul?

Täispöörde teeb minutiosuti 60 min jooksul ja läbib $2\pi \cdot 8 = 16\pi$ (cm)
10 minutiga läbib osuti $16\pi \cdot 10 : 60 = \frac{8\pi}{3} \approx 8,4$ (cm).

25 minutiga läbib osuti $16\pi \cdot 25 : 60 = \frac{20\pi}{3} \approx 20,9$ (cm).

Ülesanne 3

Kunstnik painutas seinakaunistuse aluseks traadist sektorikujulise raami. Traadi pikkus oli 10 m. Leia selle sektori pindala, kui sektori raadius on 2 m.

Kui traadi pikkus on 10 m, siis $2r + l = 10 \Rightarrow l = 10 - 2 \cdot 2 = 6$ (m).
Teame, et $l = rx$ ja $S = \frac{r^2 x}{2} \Rightarrow S = \frac{lr}{2}$. Seega $S = \frac{6 \cdot 2}{2} = 6$ (m^2).
Vastus. Sektori pindala on 6 m^2 .

Ülesanne 4

Tõesta samasust.

a)
$$\frac{2\sin\alpha - \sin 2\alpha}{2\sin\alpha + \sin 2\alpha} = \frac{1 - \cos\alpha}{1 + \cos\alpha}$$

$$\frac{2\sin\alpha - \sin 2\alpha}{2\sin\alpha + \sin 2\alpha} = \frac{2\sin\alpha - 2\sin\alpha\cos\alpha}{2\sin\alpha + 2\sin\alpha\cos\alpha} = \frac{2\sin\alpha(1 - \cos\alpha)}{2\sin\alpha(1 + \cos\alpha)} = \frac{1 - \cos\alpha}{1 + \cos\alpha}$$

b)
$$\sin^2\left(\frac{\alpha}{2} + 2\beta\right) - \sin^2\left(\frac{\alpha}{2} - 2\beta\right) = \sin\alpha \cdot \sin 4\beta$$

$$\begin{aligned}
 \sin^2\left(\frac{\alpha}{2} + 2\beta\right) - \sin^2\left(\frac{\alpha}{2} - 2\beta\right) &= \left[\sin\left(\frac{\alpha}{2} + 2\beta\right) - \sin\left(\frac{\alpha}{2} - 2\beta\right) \right] \\
 \left[\sin\left(\frac{\alpha}{2} + 2\beta\right) + \sin\left(\frac{\alpha}{2} - 2\beta\right) \right] &= \left(\sin \cancel{\frac{\alpha}{2}} \cos 2\beta + \cos \cancel{\frac{\alpha}{2}} \sin 2\beta - \sin \cancel{\frac{\alpha}{2}} \cos 2\beta + \cos \cancel{\frac{\alpha}{2}} \sin 2\beta \right) \\
 \left(\sin \cancel{\frac{\alpha}{2}} \cos 2\beta + \cos \cancel{\frac{\alpha}{2}} \sin 2\beta + \sin \cancel{\frac{\alpha}{2}} \cos 2\beta - \cos \cancel{\frac{\alpha}{2}} \sin 2\beta \right) &= 2 \cos \frac{\alpha}{2} \sin 2\beta \cdot 2 \sin \frac{\alpha}{2} \cos 2\beta = \\
 2 \cos \cancel{\frac{\alpha}{2}} \sin \cancel{\frac{\alpha}{2}} \cdot 2 \sin 2\beta \cos 2\beta &= \sin \alpha \cdot \cos 4\beta .
 \end{aligned}$$

Ülesanne 5

Arvuta avaldise täpne väärustus.

c) $2 \cos 30^\circ - 4 \cos 45^\circ + \tan 60^\circ = 2 \cdot \frac{\sqrt{3}}{2} - 4 \cdot \frac{\sqrt{2}}{2} + \sqrt{3} = \sqrt{3} - 2\sqrt{2} + \sqrt{3} = 2(\sqrt{3} - \sqrt{2})$

d) Eraldame esmalt täispöörded ja siis asendame täpsete väärustega.

$$5 \tan 540^\circ + 2 \cos 1170^\circ + 4 \sin 990^\circ - 5 \cos 540^\circ =$$

$$5 \tan 180^\circ + 2 \cos 90^\circ + 4 \sin 270^\circ - 5 \cos 180^\circ = 5 \cdot 0 + 2 \cdot 0 + 4 \cdot (-1) - 5 \cdot (-1) = 1$$

e) $10 \cot 135^\circ \cdot \sin 300^\circ \cdot \cos 135^\circ = 10 \cot(180^\circ - 45^\circ) \cdot \sin(360^\circ - 60^\circ) \cdot \cos(180^\circ - 45^\circ) =$
 $= 10(-\cot 45^\circ) \cdot (-\sin 60^\circ) \cdot (-\cos 45^\circ) = -10 \cdot 1 \cdot \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = -2,5\sqrt{6}$.

f) $\frac{\tan 135^\circ}{\tan 225^\circ} \cdot \sin^2 300^\circ - \cos^2(-300^\circ) = \frac{\tan(180^\circ - 45^\circ)}{\tan(180^\circ + 45^\circ)} \cdot \sin^2(360^\circ - 60^\circ) - \cos^2(360^\circ - 60^\circ) =$
 $= \frac{-\tan 45^\circ}{\tan 45^\circ} \cdot \sin^2 60^\circ - \cos^2 60^\circ = -1 \cdot \left(\frac{\sqrt{3}}{2} \right)^2 - \left(\frac{1}{2} \right)^2 = -\frac{3}{4} - \frac{1}{4} = -1$

Ülesanne 6

Lihtsusta avaldist.

a) $\frac{\sin \alpha}{\tan \alpha} + \cos \alpha = \cos \alpha + \cos \alpha = 2 \cos \alpha$

$$2 \cos^2 \frac{\alpha}{2} - \cos \alpha = 2 \cos^2 \frac{\alpha}{2} - \cos\left(2 \cdot \frac{\alpha}{2}\right) = 2 \cos^2 \frac{\alpha}{2} - \left(\cos^2 \frac{\alpha}{2} - \sin^2 \frac{\alpha}{2}\right) =$$

b) $= 2 \cos^2 \frac{\alpha}{2} - \cos^2 \frac{\alpha}{2} + \sin^2 \frac{\alpha}{2} = \cos^2 \frac{\alpha}{2} + \sin^2 \frac{\alpha}{2} = 1$

c) $(\sin \alpha - \cos \alpha)^2 + \sin 2\alpha = \sin^2 \alpha - 2 \sin \alpha \cos \alpha + \cos^2 \alpha + \sin 2\alpha = 1$

d) $2 \cos^2 \alpha - \cos 2\alpha = 2 \cos^2 \alpha - \cos^2 \alpha + \sin^2 \alpha = \cos^2 \alpha + \sin^2 \alpha = 1$

e) $\tan(45^\circ + \alpha) \cdot \tan(45^\circ - \alpha) = \frac{\tan 45^\circ + \tan \alpha}{1 - \tan 45^\circ \tan \alpha} \cdot \frac{\tan 45^\circ - \tan \alpha}{1 + \tan 45^\circ \tan \alpha} =$
 $= \frac{1 + \tan \alpha}{1 - \tan \alpha} \cdot \frac{1 - \tan \alpha}{1 + \tan \alpha} = 1$