

HARJUTUSÜLESANDED lk. 44

Lahenda võrratused.

1)

$$\frac{5(1-2x)}{6} + \frac{5}{8} > \frac{3x-1}{12} - 2x \quad | \cdot 24$$

$$20(1-2x) + 15 > 6x - 2 - 48x$$

$$20 - 40x + 15 > 6x - 2 - 48x$$

$$2x > -37 \quad | : 2$$

$$x > 18,5$$

2) $\frac{2x+1}{x-2} > \frac{x+4}{2x-5}$

$$\frac{2x+1}{x-2} - \frac{x+4}{2x-5} > 0$$

$$\frac{4x^2 - 10x + 2x - 5 - x^2 + 2x - 4x + 8}{(x-2)(2x-5)} > 0$$

$$\frac{3x^2 - 10x + 3}{(x-2)(2x-5)} > 0 \Leftrightarrow (3x^2 - 10x + 3)(x-2)(2x-5) > 0$$

Kasutame lahenduseks intervallide meetodit.

Leiame sulgudes olevate avaldiste nullkohad.

$$3x^2 - 10x + 3 = 0$$

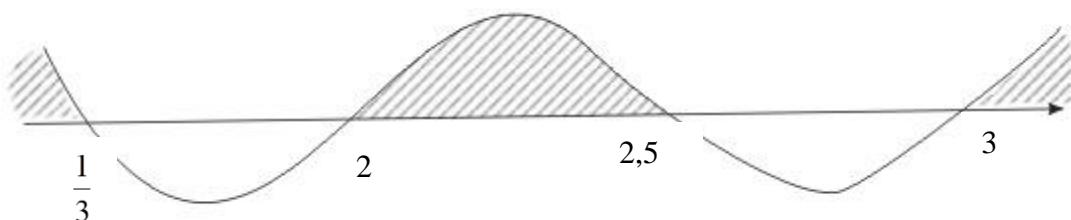
$$x_1 = 3$$

$$x_2 = \frac{1}{3}$$

$$x-2=0 \Rightarrow x=2$$

$$2x-5=0 \Rightarrow x=2,5$$

Kanname saadud nullkohad arvoteljele.



Leiame jooniselt lahendihulga $\left] -\infty; \frac{1}{3} \right[\cup]2; 2,5[\cup]3; \infty[$.

$$3) \frac{(2x-3)x^2(4-x)^3}{(x-6)^5(x^2+4x+6)} \leq 0 \Leftrightarrow (2x-3)x^2(4-x)^3(x-6)^5(x^2+4x+6) \leq 0$$

Kasutame lahenduseks intervallide meetodit.

Leiame tegurite nullkohad.

$$2x-3=0 \Rightarrow x_1 = 1,5$$

$$x^2=0 \Rightarrow x_{2,3} = 0$$

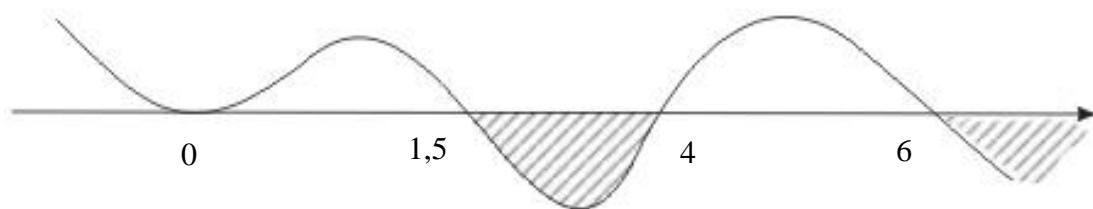
$$4-x=0 \Rightarrow x_{4-6} = 4$$

$$x-6=0 \Rightarrow x_5 = 6$$

$$x^2 + 4x + 6 = 0$$

Viimasel avaldisel nullkohad puuduvad (diskriminant on negatiivne) ja avaldis on alati positiivne.

Kanname saadud tulemused arvteljele.



Leiame jooniselt lahendihulga $\{0\} \cup [1,5;4] \cup]6,\infty[$.

$$4) x^2 - 3 < 0$$

Leiame avaldise nullkohad:

$$x^2 - 3 = 0 \Rightarrow x^2 = 3 \Rightarrow x_1 = \sqrt{3}, x_2 = -\sqrt{3} \text{ ja kanname need arvteljele.}$$

Leiame jooniselt lahendihulga $]-\sqrt{3}; \sqrt{3}[$

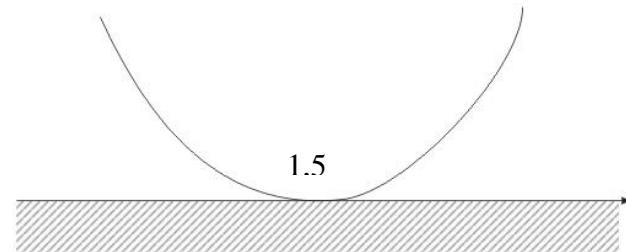
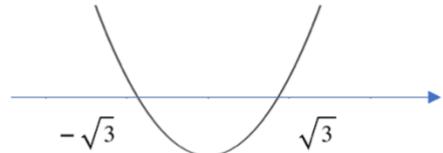
$$5) 4x^2 - 12x + 9 < 0$$

Leiame nullkohad.

$$4x^2 - 12x + 9 = 0$$

$$x_1 = x_2 = 1,5$$

Kuna parabool avaneb üles ja puutub x- telge punktis 1,5, siis võrratusel lahendid puuduvad.



$$6) (3x-1)(4-x)(2x+3)^2 < 0$$

Kasutame lahenduseks intervallide meetodit.

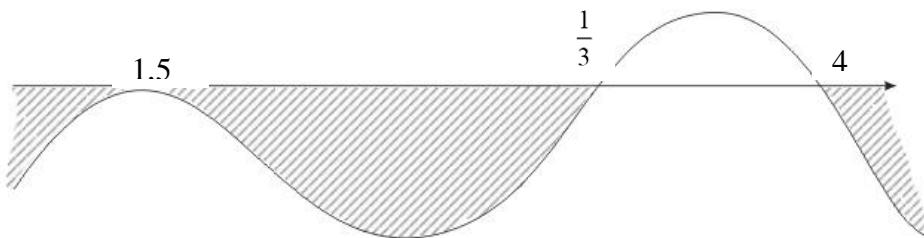
Leiame sulgudes olevate avaldiste nullkohad.

$$3x - 1 = 0 \Rightarrow x_1 = \frac{1}{3}$$

$$4 - x = 0 \Rightarrow x_2 = 0$$

$$2x + 3 = 0 \Rightarrow x_{3,4} = -1,5$$

Kanname saadud nullkohad arvteljele.



Leiame jooniselt lahendihulga $\left] -\infty; \frac{1}{3} \right[\cup]4; \infty[\setminus \{-1,5\}$.

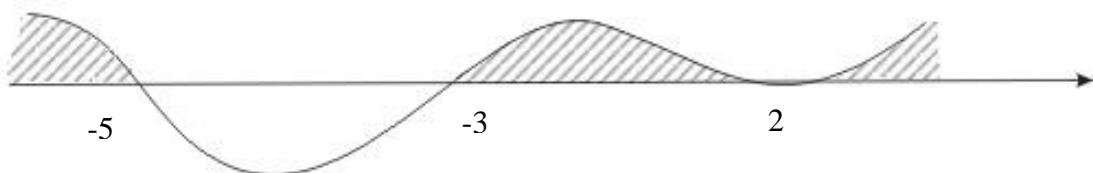
$$7) \frac{(2-x)^2(x+5)^3}{x+3} > 0 \Leftrightarrow (2-x)^2(x+5)^3(x+3) > 0$$

Kasutame lahenduseks intervallide meetodit.

Leiame sulgudes olevate avaldiste nullkohad.

$$x_{1,2} = 2, x_{3-5} = -5 \text{ ja } x_4 = -3.$$

Kanname saadud nullkohad arvteljele.



Leiame jooniselt lahendihulga $\left] -\infty, 5 \right[\cup \left] -3; \infty \right[\setminus \{2\}$.

$$8) \frac{x^2 - x - 12}{x^2 + 2x - 15} > 0 \Leftrightarrow (x^2 - x - 12)(x^2 + 2x - 15) > 0$$

Kasutame lahenduseks intervallide meetodit.

Leiame sulgudes olevate avaldiste nullkohad.

$$x^2 - x - 12 = 0$$

$$x_1 = -3$$

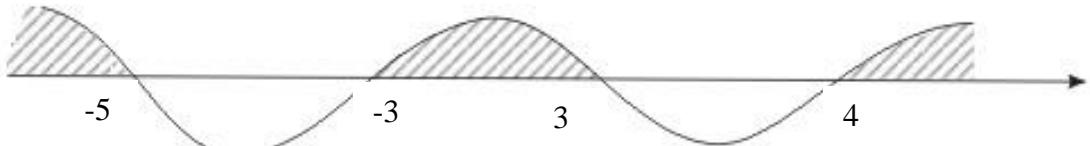
$$x_2 = 4$$

$$x^2 + 2x - 15 = 0$$

$$x_3 = -5$$

$$x_4 = 3$$

Kanname saadud nullkohad arvteljele.



Leiame jooniselt lahendihulga $]-\infty; -5[\cup]-3; 3[\cup]4; \infty[$.

$$9) \frac{2x^2 - 4x - 6}{4x - 11} \geq 2$$

$$\frac{2x^2 - 4x - 6}{4x - 11} - 2 \geq 0$$

$$\frac{2x^2 - 4x - 6 - 8x + 22}{4x - 11} \geq 0$$

$$\frac{2x^2 - 12x + 16}{4x - 11} \geq 0 \Leftrightarrow (2x^2 - 12x + 16)(4x - 11) \geq 0$$

Kasutame lahenduseks intervallide meetodit.

Leiame sulgudes olevate avaldiste nullkohad.

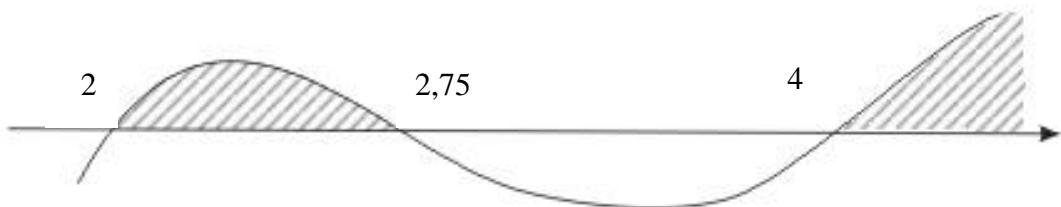
$$2x^2 - 12x + 16 = 0$$

$$x_1 = 2$$

$$x_2 = 4$$

$$4x - 11 = 0 \Rightarrow x_3 = 2,75$$

Kanname saadud nullkohad arvteljele.



Leiame jooniselt lahendihulga $[2; 2,75[\cup]4; \infty[$.

10)

$$2x^2 \geq (x - 3)^2 + (x + 2)^2$$

$$2x^2 - x^2 + 6x - 9 - x^2 - 4x - 4 \geq 0$$

$$2x \geq 13 \quad | :2$$

$$x \geq 6,5$$

$$11) 2 - \frac{5-x}{7} < 1 - \frac{9-x}{14}$$

$$2 - \frac{5-x}{7} - 1 + \frac{9-x}{14} < 0 \quad | \cdot 14$$

$$28 - 10 + 2x - 14 + 9 - x < 0$$

$$x < -13$$

$$12) \begin{cases} \frac{x-4}{5} + \frac{x-3}{7} \leq 0 \\ x - \frac{2}{3}x \geq \frac{1}{3} \end{cases}$$

Lahendame mõlemad võrratused eraldi ja leiame siis nende ühisosa.

$$\frac{x-4}{5} + \frac{x-3}{7} \leq 0 \quad | \cdot 35$$

$$7x - 28 + 5x - 15 \leq 0$$

$$12x \leq 43 \quad | :12$$

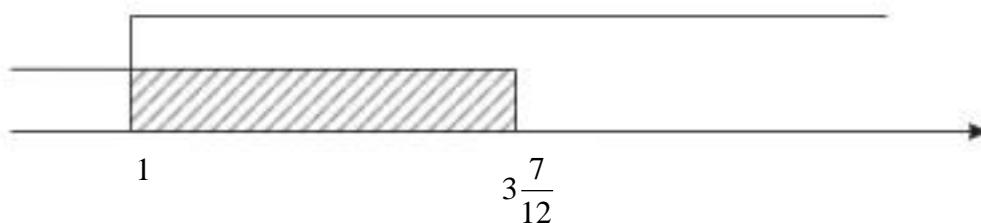
$$x \leq \frac{7}{12}$$

$$x - \frac{2}{3}x \geq \frac{1}{3} \quad | \cdot 3$$

$$3x - 2x \geq 1$$

$$x \geq 1$$

Kanname mõlemad lahendihulgad joonisele ning leiame nende ühisosa.



Võrratussüsteemi lahendihulgaks on $\left[1; 3\frac{7}{12}\right]$

$$13) (x+3)(x-1)(x-2) < 0$$

Kasutame lahenduseks intervallide meetodit.

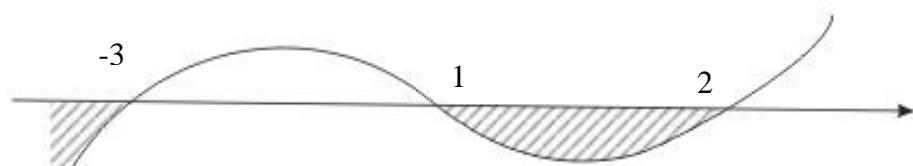
Leiame sulgudes olevate avaldiste nullkohad.

$$x+3=0 \Rightarrow x=-3$$

$$x-1=0 \Rightarrow x=1$$

$$x-2=0 \Rightarrow x=2$$

Kanname saadud nullkohad arvteljele.



Leiame jooniselt lahendihulga $]-\infty; -3] \cup]1; 2[$

$$14) \frac{3x+2}{2x+1} > 5$$

$$\frac{3x+2}{2x+1} - 5 > 0$$

$$\frac{3x+2-10x-5}{2x+1} > 0$$

$$\frac{-7x-3}{2x+1} > 0 \Leftrightarrow (-7x-3)(2x+1) > 0$$

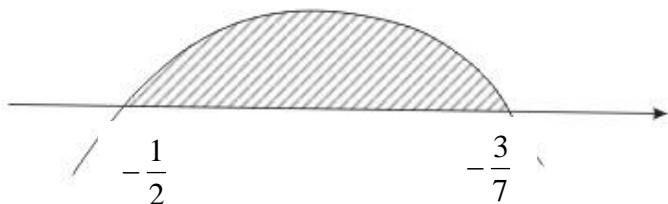
Kasutame lahenduseks intervallide meetodit.

Leiame sulgudes olevate avaldiste nullkohad.

$$-7x-3=0 \Rightarrow x = -\frac{3}{7}$$

$$2x+1=0 \Rightarrow x = -0,5$$

Kanname saadud nullkohad arvteljele.



Leiame jooniselt lahendihulga $\left] -\frac{1}{2}; -\frac{3}{7} \right[$

$$15) \begin{cases} \frac{x+3}{x-2} < 1 \\ \frac{2x+3}{3x-2} < 2 \end{cases}$$

Lahendame mõlemad võrratused eraldi ning leiame siis ühisosa.

$$\frac{x+3}{x-2} - 1 < 0$$

$$\frac{x+3-x+2}{x-2} < 0$$

$$\frac{5}{x-2} < 0 \Rightarrow x-2 < 0 \Rightarrow x < 2$$

Teise võrratuse lahendihulga same

$$\frac{2x+3}{3x-2} - 2 < 0$$

$$\frac{2x+3-6x+4}{3x-2} < 0$$

$$\frac{-4x+7}{3x-2} < 0 \Leftrightarrow (-4x+7)(3x-2) < 0$$

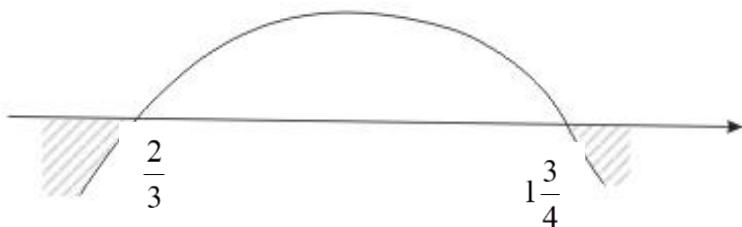
Kasutame lahenduseks intervallide meetodit.

Leiame sulgudes olevate avaldiste nullkohad.

$$-4x + 7 = 0 \Rightarrow x_1 = 1\frac{3}{4}$$

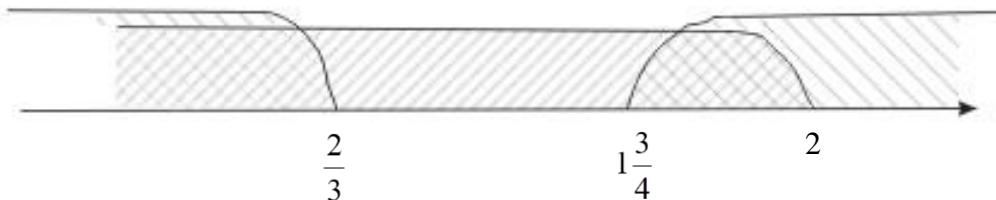
$$3x - 2 = 0 \Rightarrow x_2 = \frac{2}{3}$$

Kanname saadud nullkohad arvteljele.



Leiame jooniselt lahendihulga $\left] -\infty; \frac{2}{3} \right[\cup \left] 1\frac{3}{4}; \infty \right[$.

Leiame lahendipiirkondade ühisosa.



Saime tulemuseks $\left] -\infty; \frac{2}{3} \right[\cup \left] 1\frac{3}{4}; 2 \right[$

16) $|x^2 + 2x - 3| = 5$

a) $x^2 + 2x - 3 = 5$

$x^2 + 2x - 8 = 0$

$x_1 = -4$

$x_2 = 2$

b) $x^2 + 2x - 3 = -5$

$x^2 + 2x + 2 = 0$

Sellel võrrandil lahendid puuduvad (diskriminant on negatiivne).

Lahendite kontroll teosta iseseisvalt.

Vastus. Antud võrrandi lahenditeks on -4 ja 2 .

17) $|2x^2 - 8x| = -3$ lahendid puuduvad, kuna absoluutväärust ei saa olla negatiivne.

18)

$$|3x - 2| = |1 - 3x|$$

$$a) \quad 3x - 2 = 1 - 3x$$

$$6x = 3 : 6$$

$$x = 0,5$$

$$b) \quad 3x - 2 = -(1 - 3x)$$

$$3x - 2 = -1 + 3x$$

$$0 = 1$$

Saime vastuolu, seega sellel võrrandil lahendid puuduvad.

Kontroll:

$$|3 \cdot 0,5 - 2| = |1,5 - 2| = |-0,5| = 0,5$$

$$|1 - 3 \cdot 0,5| = |1 - 1,5| = |-0,5| = 0,5$$

$$vp = pp$$

Lahend: $x = 0,5$.